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Minimization Principle for the Unit of Electric Charge

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The fine-structure constant is given theoretically as $\alpha = (137.03608)^{-1}$ if the eigenvalue of a charge-related operator is minimal for $\pm e/3$ quark states.

Recent work by Morris (1980) has shown that the fine-structure constant cannot have a dynamical origin in the context of nonlinear field theory, as suggested as a possibility many years ago (Rosen, 1939). Thus one is led to seek an internal kinematic principle which fixes the physical value of α.

Under electromagnetic gauge transformations $a_\mu \to (a_\mu + \phi,_\mu)$, the wave function of a quark or charged lepton transforms $\Psi_\varepsilon \to (\exp i\varepsilon\phi)\Psi_\varepsilon$ with the charge $\varepsilon = \pm e/3$, $\pm 2e/3$, $\pm e$. The wave function can be expressed as

$$\Psi_\varepsilon = (\exp 2\pi i \varepsilon \xi)\psi(x) \tag{1}$$

where $\psi(x)$ is a gauge-invariant Dirac spinor and the scalar field $\xi = \xi(x)$ transforms additively under gauge transformations: $\xi \to [\xi + (1/2\pi)\phi]$. Observe that Ψ_ε shown in (1) is an eigenstate of

$$D \equiv -2\frac{\partial^2}{\partial \xi^2} \tag{2}$$

In terms of the latter operator, the physical value of α obtains from the condition that the eigenvalue of

$$\left(D^5 - \frac{2\pi}{3}D\right) \tag{3}$$

be minimal for quark states corresponding to the smallest unit of charge, $\varepsilon = \pm e/3$.

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Proof of the Minimization Principle. From (1) and (2) it follows that $D\Psi_\varepsilon = \lambda\Psi_\varepsilon$ with $\lambda = 8\pi^2 e^2/9$ for $\varepsilon = \pm e/3$. Given by $[\lambda^5 - (2\pi/3)\lambda]$, the eigenvalue of (3) is a minimum[1] for $d/d\lambda\ [\lambda^5 - (2\pi/3)\lambda] = 0$, i.e., $\lambda = (2\pi/15)^{1/4} = 0.80449278$. Hence, one obtains

$$e^2 = \frac{9\lambda}{8\pi^2} = (10.904985)^{-1} \tag{4}$$

and

$$\alpha \equiv \frac{e^2}{4\pi} = (137.03608)^{-1} \tag{5}$$

More precisely, the minimization principle gives the fine-structure constant $\alpha = (9/32\pi^3)(2\pi/15)^{1/4}$.

REFERENCES

Morris, T. F. (1980). *Physics Letters*, **93B**, 440.
Rosen, N. (1939). *Physical Review*, **55**, 94.

[1]Since (2) is the square of the self-adjoint operator $i\sqrt{2}\,\partial/\partial\xi$, the eigenvalue spectrum of (2) is real and non-negative; thus negative (or imaginary) values of λ are precluded.